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14.9. Chinese Remainder Theorem :- Let $m_1, m_2,$ ---- m_r be pairwise relatively prime numbers such that $\gcd(m_i, m_j) = 1$ for $i \neq j$. Then the system of linear congruences

$$x \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$
$$\vdots$$
$$x \equiv a_r \pmod{m_r}$$

has a simultaneous solution which is unique modulo $m_1, m_2,$ ---- m_r.

Let $m = m_1 m_2 \ldots m_r$ and let

$M_k = \frac{m}{m_k}$.

$\gcd(M_k, m_k) = 1$

Ex:- Solve the system of of linear congruence.

$$x \equiv 3 \pmod{11}$$
$$x \equiv 5 \pmod{19}$$
$$x \equiv 10 \pmod{29}$$

Here $m = 11 \cdot 19 \cdot 29 = 6061$

$a_1 = 3, \ a_2 = 5, \ a_3 = 10, \ m_1 = 11, \ m_2 = 19$

$m_3 = 29$

Now $M_1 = \frac{m}{m_1} = \frac{6061}{11} = 551$

$M_2 = \frac{m}{m_2} = \frac{6061}{19} = 319$

$M_3 = \frac{m}{m_3} = \frac{6061}{29} = 209$

Now the linear congruences

$551x \equiv 1 \pmod{11}$
$319x \equiv 1 \pmod{19}$
$209x \equiv 1 \pmod{29}$

are satisfied by $x_1 = 1, x_2 = 14$ and $x_3 = 5$ respectively.

A solution of the system is given by

$\overline{x} = a_1 M_1 x_1 + a_2 M_2 x_2 + a_3 M_3 x_3$

$= 3.551.1 + 5.319.14 + 10.209.5$
$= 1653 + 22330 + 10450$
$= 34433$
$= 4128 \pmod{6061}$

is the unique solⁿ by Ch- remainder theorem.

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M.9 Solvability condition of a system of Linear Congruences.

A system of linear congruences

$x \equiv a_1 \pmod{m_1}$

$x \equiv a_2 \pmod{m_2}$

$\vdots$

$x \equiv a_r \pmod{m_r}$

is solvable if (m_i, m_j) divides $(a_i - a_j)$

Ex:- Is the following system solvable?

$x \equiv 2 \pmod 5$

$x \equiv 4 \pmod{21}$

$x \equiv 3 \pmod{14}$

Sol:- Here we have.

$4-2=2$ is divisible by $(21,5)=1$

$3-2=1$ " " " $(14,5)=1$

$4-3=1$ " not " " as $(21,14)=7$

Hence the given system is not solvable.